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Teiji Takagi: a biography

—On the 100th anniversary of his birth—

by

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Teiji Takagi (1875-1960) was the first Japanese mathematician to achieve worldwide fame. In the years of the First World War, he discovered that every relatively Abelian field K over any algebraic number field k is a class field over some ideal class group in k. Founded on this main theorem, he created a magnificient edifice of his class field theory, in the utter (scientific) solitude engendered by the World War. His two famous papers (1920, 1922) after the War gave strong stimulation to several young European mathematicians (Artin, Hasse, Chevalley, and others). Today, the scientific level of a book of algebraic number theory can roughly be estimated by whether it contains class field theory or not, though its modern theory, using homological methods, is greatly different in appearance from Takagi-Artin's classical one.

The author intends to publish a detailed biography of Takagi in Japanese in a few years. Since his life seems to be scarcely known abroad, it will be described in this article mainly for foreign mathematicians, on this very occasion of the 100th anniversary of his birth.

I. Three generations of Kansuke—heavy burdens on the boy Takagi's back

Three generations of Kansuke

Kôshôji, which has always been the family Buddhist temple of Takagi's native home, is situated about 8 kilometers west from Gifu City. The author visited this temple in the middle of October, just at a time when many, many kaki (persimmon) trees bearing luscious fruit, for which this district of Gifu Prefecture is quite famous, could be seen everywhere around the temple. Going through a red temple gate, the author met the Rev. Mr. Shimono, the chief priest, in a room of the main temple. Occasionally, a snake could be seen creeping in the tokonoma (Japanese alcove). Mrs. Shimono expelled it from the room, smiling, with the explanation: "Since there is a thicket in the rear of our house, ···." In order to write a biography of a person,

one may possibly even encounter a snake!

The Rev. Mr. Shimono brought there several old documents, and began to investigate the ancestors of the Takagis. For instance, there was such a sentence as: 'Kansuke of Kazuya Village died on December 16 in 1812, with the posthumous name Chisei.' This person Kansuke was Kansuke the First, an ancestor of Teiji Takagi.

After the investigation of the documents, we were reposing leisurely, and the refreshing sough of the wind in the bamboo thicket in back and the harmonious twitterings of birds went straight to the author's heart, since he was a visitor from a smoggy, great city. "What kinds of birds come here?," the author asked the Rev. Mr. Shimono. The priest, also relaxed, answered. "Shrikes, (Japanese) buntings, tits, pheasants, and (Japanese) bush warblers come here. And starlings, hundreds of them, come in a large group." He concluded with a laugh, "However, the birds which are most numerous here are, after all, sparrows."

In order to go to the native house of Takagi from this temple, one must go through Nakanomoto and Kurono, and then cross the Yabu River Bridge. It is a distance of about 5 kilometers.

The present address of the native house is Kazuya 557, Itonuki-machi, Motosu-gun, Gifu Prefecture. The house has the appearance of quite an ordinary Japanese farmer's house. Inside the entrance, there is an earthen floor, and in the right corner of it, a cow is now being kept. However, when Takagi was born, the domestic animal was a horse. It was still a wonder for the author that this mathematician of international fame in later years was brought up in the same building with a horse! Towards the left side from the earthen floor, there are some rooms, one of which faces north. This is the very room where Takrgi was born. On its wall, there hang a spear and two wooden swords, which were used by Kansuke the Third, the foster father of Takagi. The author felt a peculiar sensation when he thought that the magnificient class field theory had started from this dim and somewhat wretched room.

Now we will state the history of the Takagis, according to the documents of Kôshôji and the talks of some old men in Kazuya.

There is no genealogical table of the Takagis. This family seems to have been quite ordinary farmers, having no eminent ancestors. However, Kansuke the First, whom we wrote of before, was a clear-headed man, and became very rich by brewing. Since he owned an unusually vast farm, there remains a legend about him that he could go all the way from his house to a place at some distance, called Tokino, on the Ibi Highway without ever leaving his own estate. His son and grandson succeeded to the name Kansuke. This fact also

shows that he was the first person of distinction in the history of the Takagis.

But the life of Kansuke the Second was not so admirable as his father's. It seems that he was not completely incompetent, for he did become the Chief of Kazuya Village. Nevertheless, he gradually lost his property which his father acquired by many a toil. (The reason is not very clear.) Finally, he was ruined, and, leaving his family in Kazuya, he went to Ôsaka, where he became a clerk in Shitenôji (a very famous Buddhist temple). He died there in 1853.

His wife, Taka, was alive until 1895, when Teiji Takagi was twenty years old. Kansuke the Second and Taka had two sons and a daughter. The elder son was Kansuke the Third, born in 1835. The younger was Genjûrô, who died young. The daughter, mother of Teiji, was Tsune, born in 1843.

Hereafter, we shall call Kansuke the Third simply as Kansuke. When he was six years old, his home was financially ruined, and a creditor had his house torn down. The daring boy got angry, and cried to the creditor. "Why did you have my house torn down?" His mother then explained to him that the house was no longer their property, having been mortgaged for the debt. Kansuke, though he was a little boy, listened well to reason. However, owing to this sad experience in his boyhood, he grew up later into an extremely austere and hardworking man.

After this ruin, he worked diligently in another person's home. About fifteen years later, Kansuke recovered his property to a certain extent. He married Iwo, his cousin, who brought him a girl from her former marriage. However, Kansuke could not have any children of his own by Iwo. This circumstance was to have a close connection with Takagi's life.

The leeches

About 1875, by the time Takagi was born, Kansuke Takagi had amassed fortune with a tenant farm, while his sister Tsune, mother of Takagi, worked on a farm with one other worker under her. Kansuke himself was the treasurer in the village office of Kazuya. He seems to have been very efficient in his work. Tsune was a spirited and frank woman with a very keen memory. For instance, she could remember well the accurate date when even a trifling thing happened.

In 1874, Tsune married Mitsuzô Kinomura of the Town of Kitagata. For Mitsuzô this was his second marriage, and he was then fifty-five years old, while Tsune was thirty-one. He was the owner of a farm, but was also employed in the town office. He is also said to have had a clear head.

We shall now relate the 'story of the leeches':

Soon after the marriage, Tsune became pregnant, and went back to her native home for delivery. On April 21 in 1875, she gave birth to Teiji Takagi. However, she never returned to Mitsuzô Kinomura—and the reason was her dislike for leeches!

In general, leeches are abundant in such paddy fields that are located low and contain stagnant water. Since the fields of the Kinomuras were also located low, they were filled with leeches. Leeches stick to the hands and legs of farmers working in paddy fields, and suck blood. It seems that old Japanese farmers resigned themselves to this hazard as if it were inevitable when working in fields. But Tsune disliked leeches violently. She, even after the birth of Teiji, did not return to her husband's home. Her brother Kansuke reported Teiji to the village office as the son of himself and his wife (Iwo).

We can guess the intention of Kansuke: it was a glorious legend of the Takagis that Kansuke the First could go all the way to Tokino just through his own estate. However, his son, Kansuke the Second, was ruined. It was perhaps a strong desire of Kansuke the Third to revive the legendary glory of his grandfather. But, since he had no son, he had to entrust the realization of the dream to Teiji, his nephew. And, in fact, Teiji later accomplished a far better thing than his foster father desired. We can say that the misfortune of Kansuke the Second was fully made up for by his grandson.

It is interesting to note that Mitsuzô Kinomura did not send his own two sons to universities, and so, if Tsune had remained his wife, it is not improbable that Teiji could not have become a mathematician. Thus, there may be a queer relation between the leeches in the paddy fields of the Kinomuras and the history of Japanese mathematics!

Private school of Shôtô Nogawa

When Teiji was born, he was a rather little baby, and was not very healthy even afterwards. Kansuke, as well as Tsune, brought him up with deep affection.

In summer, the children of Kazuya swam pleasantly in the Yabu River. But Teiji did not swim. He was brought up as if he were a doll, being kept from any kind of play with the slightest sign of danger.

His mother Tsune was a very pious Buddhist, and very much liked worshipping at temples, together with Teiji. Now, there is a Buddhist scripture named Godenshô, which is rather long. One day, Teiji of age about five astonished the grown-up people by reciting Godenshô by rote.

Next to the native house of Takagi, there lived a doctor called

Kyôhei Nogawa. He was the very person who gave Teiji the first cultural impetus. He had a pen name of Shôtô Nogawa, and his many accomplishments included Chinese literature, calligraphy, Chinese painting, flower arrangement, and tea ceremony.

Takagi entered Nogawa's private school at the age of about five, and studied Chinese literature (the Analects of Confucius, Mencius, and others). His later style of writing was succinct with a lofty tone, and it seems to have been influenced much by the study of Chinese literature in his boyhood.

Kyôhei Nogawa had two sons and five daughters. Concerning Tazuko, the third daughter, we shall have more to say later.

Isshiki School

In 1882, Teiji at the age of seven entered Isshiki School, the unique primary school of the Village of Kazuya. He achived exceedingly good results, and, skipping grades, for instance, from the first grade to the third grade, he finished the regular six years' course in only three years. Then he advanced to the higher course.

The village office, where Kansuke worked, adjoined the school; and after school, Teiji used to go to the office, without playing with the other children. He studied at the desk next to Kansuke's. Parents in the village often scolded their children harshly as follows. "Don't always be playing. Be like the son of Mr. Treasurer!" Therefore, the neighborhood children were much troubled by this boy, Takagi.

Yau Umeda, a girl who was in the same class as Teiji, said to a certain person in later years: "Since he was such a lovely little boy, I wished to touch him. However, on the other hand, since he had such a distinguished record, and rarely talked to anyone, he was indeed one boy who was difficult to approach."

Teiji's brilliancy in school was even reported in a newspaper. The following article was printed in the Gifu Daily News on January 7 in 1886:

'Teiji, the eldest son of Kansuke Takagi in the Village of Kazuya, Motosu-gun, though now only ten years and a month in age, is already in the higher third-year class of Isshiki Primary School. In addition to his excellent qualities, he is quite diligent. Moreover, his teachers and his father teach him very kindly. Thus he advances rapidly in his study. He is indeed a genius, for whose future we have great expectations.'

However, there was a girl who contended with him for the top in their class of Isshiki School. It was Tazuko, the third daughter of the Nogawas.

Though she was a pretty girl, she was strong-minded and romped

as if she were a boy. She was skillful in walking on walls and in tree climbing. As we wrote before, this district is famous for the fruit of kaki. In her family, it was Tazuko's part to climb the trees of kaki, and to pick the fruit. However, her school records were very good. When Teiji was the head of his class, she would be second, and then when she was the head, Teiji would be second. When he was the second due to her, this weak boy was made to stand outdoors, with a heavy desk on his back. It was Kansuke's punishment. The heavy burden on the boy's back seems to have been a symbol of the three generations of Kansuke.

II. Nyorai

Gifu-Prefectural Ordinary Middle School

In 1886, Teiji Takagi entered the Gifu-Prefectural Ordinary Middle School, which was in the Town of Gifu.

Takagi himself wrote later about this period as follows:

'In those days, there seems to have heen no Japanese text-books, and we used only foreign books. Thus the text-books of English were readers of the United States. What worried us most was the spelling lesson in the first-year. Young people of today perhaps don't know such lessons; it aimed at teaching orthography to American pupils. They learn by it the spellings of words, whose meanings they know quite well. But in our case, on the contrary, we had to give the answer for the spellings of words by hearing their pronunciations, without the knowledge of their meanings! The teacher pronounced "baker". Then we had to answer "b-a, ba, k-e-r, ker, baker". According to each student's record in memorizing several tens of words in one lesson, our seating order in the spelling class was changed every time.'

(Concerning the foregoing quotation, we would like to add a word of explanation for foreign readers: after the Meiji Restoration in 1868, Japan introduced modern European education. The year 1886, when Takagi entered the middle school, was only 18 years after the Restoration, and Japanese education was still in the very early stage of Europeanization.)

In Takagi's school, there was a teacher of mathematics named Jûrô Teshima. He gave very difficult problems in examinations. However, only Takagi solved them easily. The other pupils complained of Teshima's practice of giving problems as if he wanted to test the ability of Takagi.

On March 31 in 1891, Takagi graduated first on the list from this middle school.

Third Higher Middle School

In September of 1891, Takagi entered the Third Higer Middle School in Kyôto. Kansuke and Tsune brought big loads on their shoulders from Kazuya to Gifu Station, from where Teiji started for Kyôto by railway.

However, soon after his entrance into the school, a huge disaster struck the district of Gifu. On October 28 of 1891, the Great Earthquake of Nôbi occurred! For instance, in Gifu City, 230 men died, 906 houses collapsed, and 2017 houses were burnt.

Takagi, learning of this earthquake, felt much anxiety about his family. He went as far as Maibara by railway, and then, since the rail service beyond there was stopped, he walked and walked on foot all the way to his native house in Kazuya.

Fortunately, safe were his family and the main house except a detached room which collapsed. However, Kansuke scolded Teiji violently. "You have an important task to study. There is no need to fear about your family. Go back to Kyôto immediately!" Mother Tsune tried to mediate so as to give her son lodging at least that night. But Kansuke made Teiji return at once. Such a person was he, Kansuke Takagi! Fifty years later, Teiji said to a friend that he had been able to succeed in mathematics just because Kansuke forbade him to enter the house on that occasion.

Now it was in this period of the Third Higher Middle School when he decisively aspired to become a mathematician. Professor Jûtaro Kawai of this school influenced him most towards that direction. Graduated from Tôkyô University, Kawai went to Germany for study and attended the lectures of F. Klein and H. Weber. His speciality was function theory. In short, he was the first researcher of mathematics, not a mere teacher of it, whom Takagi had ever met. He was the subject of many interesting anecdotes, one of which goes as follows. One day while he was walking along a road, reading a book of mathematics, he walked right into a light pole. One of his disciples, who happened to witness the scene, was astonished to see his master staying right on the spot for quite a while, still intensely reading the book.

In this higher middle school, too, Takagi's records were excellent. For instance, he was the head in English, among all the same year pupils.

Imperial University

Tôkyô University was founded in 1877, ten years after the Meiji Restoration. In 1886, the name was changed to the Imperial University. Takagi entered it in July of 1894.

In the Department of Mathematics, there were only two professors—Dairoku Kikuchi and Rikitarô Fujisawa. They were the first two central figures of mathematics in modern Japan. (Before the Restoration, there was in Japan a special kind of mathematics called *wazan*, which had been expelled by European mathematics in the Meiji Era.) Kikuchi went to England for study when he was only eleven years old, and introduced into Japan mathemtics for school education. Fujisawa studied mathematics as science in Germany, and learned further to 'research'. He wrote some papers on elliptic functions. Thus we can call him the first research worker in modern Japanese mathmatics.

Now, a month or so after Takagi's entrance, Fujisawa gave an examination on calculus. Before the exam, Kôtarô Honda, who was a student of the Department of Physics, and who later became an eminent physicist, said to his friends as follows: "I have read my notebook four times. Therefore, I can do any problem!" At that time, Takagi remarked, smiling a little ironically, "Is mathematics a science to be memorized?" This irony shows his modern, intellectual character very clearly. We might add that his record in this examination was 140, the full marks being of course 100; Fujisawa represented by his own violent way his acknowledgement of Takagi's mathematical ability.

Now we shall present again that clever, but tomboyish girl—Tazuko Nogawa—who was skillful in walking on walls and in tree climbing. It seems that Takagi had cherished a sort of affection toward her—the girl who contended with him for top in the class. When he was a student in the university, he proposed to her, perhaps through Kansuke and Tsune. The Nogawas, however, refused the proposal. The cause seems to have been the ruin of Kansuke the Second.

Later, Tazuko got married to Tokusuke Kanno, a scholar of English literature. After her husband died rather young, she kept a beauty parlor with such success that it became one of the most famous parlors in Tôkyô. One day, after the Second World War, when Takagi once happened to have an interview with a radio announcer, Tazuko heard him, and joyfully, she cried to her family. "Teiji is now on the radio!" She listened to it quite longingly. We don't know exactly whether or not her parents had ever informed her of Takagi's proposal of marriage.

But let us return to the matters in the university. Takagi's nickname among his friends was Nyorai (Buddha), and it is said that this nickname signified the excellency of his brain. However, it seems to contain a bit of irony; for Takagi was silent, not sociable, and difficult of approach. When he was studying in the dormitory, he kept a fine posture, not changing it for a long time. We can feel in this nickname a certain cynicism for this 'impression of a statue of Buddha'.

Departure as a mathematician

When Takagi was in the third year, Fujisawa was the only professor in the Department of Mathematics. (Kikuchi had become the university chancellor.) In Fujisawa's seminar, Takagi studied Abelian equations. Since his class field theory was nothing other than the theory of relatively Abelian number fields, this theme of the seminar had a slight connection with his later achievement.

On July 10 in 1897, Takagi graduated from the university. The other graduates of the same department were Tsuruichi Hayashi and Takuji Yoshiye. These two had also studied under Kawai in the Third Higher Middle School.

Takagi then went on to the graduate school. In this period (about a year), he published two books of mathematics—*New Arithmetic* and *New Algebra*. In the preface of the former, he wrote in high spirits as follows:

'Concerning the word "arithmetic", there are two sorts of meaning. One is mainly relevant to the methods of counting by decimals, and complies with the demand of common society. Thus it is taught in primary schools. The other concerns the properties of integers, and is one of the most transcendental fields in mathematics. Now the contents of this book belong to the former one. However, we of course don't intend to use this for a text-book of primary schools, and there must be a certain difference in appearance between this book and the other books of arithmetic.・・・'

New Arithmetic, which was published in May of 1898, contained elementary number theory and a strict presentation of the notions of numbers (integers, fractions, and irrationals). Especially, the theory of irrationals was explained in detail by both the two methods of Heine and of Dedekind. It is true that Takagi wrote the book with reference to some European books, but it rose to an unusually high level in Japanese mathematics of that time.

The contents of *New Algebra* were polynomials, algebraic equations, determinants, and quadratic forms. It seems that Takagi wrote it mainly upon the basis of H. Weber's *Lehrbuch der Algebra*. At any rate, it is sure that he learned algebra regularly from Weber.

Although he did complete writing the manuscript of *New Algebra*, he had to leave for Germany before he could read the proofs, and so he asked one of his friends to do so for him. He wrote the preface on the steamer which was then sailing on the Indian Ocean. The book was really published in November of 1898, when he was already in Berlin.

III. Berlin, where horse-cars were noisily running

Sailing by the Armand Béhic

In May of 1898, Shôichi Toyama, Minister of Education, resigned from his post. Just before the resignation, he made public all the names of those who should be sent abroad for study in that year. Takagi's name was on the list. In those days of Japan, it was the highest honor for young men to be sent abroad by the government.

Takagi was delivered the following writ from the Minister of Education:

'You are ordered to go to Germany in order to study mathematics for three years.'

He retired from the graduate school, and started abroad on August 31. In the train to Yokohama, his master Fujisawa, who was somewhat of a gourmet, spoke to him. "Leaving now, you will reach Marseilles just in the season of mackerel of the Mediterranean Sea. It is called in French maquereau. You should taste it by all means!"

Of course, Kansuke and Tsune, too, saw him off at Yokohama. And it was the last time when Takagi saw his foster father, since the latter died before his return.

In those days, there was no direct line between Japan and Europe. Thus, he sailed by a Japanese steamer as far as Shanghai, where he then changed ships, embarking on the Armand Béhic of the Messageries Maritimes.

Since it took about forty days to go to Europe from Japan, it seems that Takagi arrived in Marseilles perpaps around October 10. Differing from travel by jet planes of today, sail by steamers was quite slow-going. A ship had to dock in Hongkong, Singapore, and many other harbors. In every one of them, the scenery, the people's clothes, the characteristic of their looks and their languages were very different from those of the other harbors'. That gave a colourful taste to sea travel. A Japanese of those days could go to Europe only through such a elaborate process. Morover, to the eyes of Japanese young men of that time, professors of European universities were something like gods! (It was before the Russo-Japanese War of 1904–1905, and Japan was still a small country in the Far East.) Thus, Takagi landed at Marseilles with excitement. He hurried to get a train for Berlin, forgetting completely Fujisawa's advice about mackerel. It was on October 13 that he arrived in this capital of Germany of monarchial days.

Schumann Street 18

The population of Berlin in 1898 was about 1,800,000; and horse-

cars and about 6,500 carriages for hire were noisily running in the city. It happened to be the eleventh year of the reign of Kaiser Wilkelm II.

Takagi lodged at Schumann Street 18. From the building, it was about fifteen minutes' walk to Berlin University (today, called Humboldt University). These belong now to East Berlin.

The building is now being used as a small medical school, and the inner parts seem to have been much changed from those when Takagi lodged there. But the outside walls and the doorway look very blackened, and still have the atmosphere of the 19th century. The image of the Holy Mother in a recess has a naive air. Since Takagi often used to go to theaters in Berlin, we can imagine such a scene as his coming back late at night from a theater, looking at the image, and then feeling vividly the relaxed sensation of returing to his own 'roost'.

In this district of Schumann Street, there is the German Theater. According to the map of that time, there were also barracks. To us Japanese, it is very queer that, opposite to a theater, there were barracks. Perhaps it might have been Prussian taste.

Finally, we shall write a bit about horse-cars, which appear often in photographs of those days. This type of car is an intermediate between a carriage and an electric car; it runs on rails, drawn by one or two horses. They were also used in the Japan of that time, which was the end of the period when they were being used in Berlin. The fare was ten Pfening minimum, while that of carriage was one Mark.

Study in the university

In Berlin University, Takagi attended the lectures on algebra by Frobenius, on number theory by Hensel, on function theory by Schwarz, on differential equations by Fuchs, and certain other lectures as well.

In his later essay *Recollections and Views of the Future*, Takagi wrote as follows:

'Fuchs and Schwarz were then so old that, if they had been living in the Japan of today, they would have had to retire by reason of having exceeded the age limit. I attended the lecture on differential equations by Fuchs, which was nothing other than the contents of his paper in Crelle's journal Vol. 66. However, for instance, the circle of convergence of a solution, which should reach the irregular point, did not reach so on the blackboard, to his embarrasment.

Frobenius was the youngest among the professors, and lectured on Galois theory and number theory. Though the content of his lectures was ordinary, his attitude was quite lively, as he lectured without the use of any notes. I had never before attended such a truly lively lecture. But, to tell the truth, I had been in some awe of Frobenius.

The circumstances were as follows: just before my departure for Germany, several young professors of the Faculty of Science of the Imperial University returned home from that country, and I listened to their talks about various things in Germany. One said to me, "If you go to Frobenius, you must be extremely careful." According to him, when Frobenius became the Dean of the Faculty, he delivered an inaugural address, and in it he glorified the progress of German science. "In order to study science in Germany, there come to this country many foreigners—Americans, and so on, and lately even Japanese! In the future, no doubt apes will be coming.···" I am doubtful about whether it was true, since it was in a public inaugural address. Perhaps it was a bit exaggerated. At any rate, I was thus warned to go to Frobenius with desperate resolve, since he disdained Japanese so much. However, when I came in actual contact with him, I discovered to my surprise that he was not such a dreadful person. Once I visited him to consult on a certain mathematical problem. The master said to me. "That is quite interesting. Denken Sie nach!" He even lent me several reprints of papers. This "Denken Sie nach" was one lesson I had never been given before.'

The contents of the lectures Takagi heard were not much more than what he had learned in Japan. However, his study in Berlin was not a waste of time, because he got used to the German language, manners, and customs. It can be viewed, by us of today, as a preparation for his study in Göttingen.

Takuji Yoshiye, a friend of Takagi in the Third Higher Middle School and the Imperial University, went to Germany in August of 1899. After a short stay in Takagi's lodging, he began to study in Göttingen. There still remain nine picture cards mailed from Takagi in Berlin to Yoshiye in Göttingen. In one of them, Takagi writes: 'It is just one year today since I arrived in Berlin. How fast time flies! I am very much ashamed of the lack of progress in my study.' Thus feeling impatience, he often shows in the post-cards his concern for Felix Klein and David Hilbert in Göttingen.

In the spring of 1900, after three semesters in Berlin University, he started for Strasbourg to attend the lectures by Weber. On the way, he dropped in at Göttingen, where his friend Yoshiye was studying. However, by his sensitive susceptibility, Takagi felt strongly the unusual mathematical ability of Hilbert, and changed his schedule to a long stay in Göttingen. This proved to be a fateful change of destination for him.

The morning bell-ringing in Göttingen

In the City of Göttingen, birds were quite abundant, and they

began to twitter in the breaking dawn. At seven o'clock in the morning, the large clock of the City Office proclaimed the time, and then immediately, the bell of the adjoining Johannis Church rang for a while with a grave, undulating tone.

This city was an important member of the Hansa League, but, after the Thirty Years' War, it declined markedly. It revived again in 1734, when George II established here a university. Thus, this is a so-called university town, and has a small population. According to the *Göttingen Chronicle*, its population became 30,185 in 1900, exceeding 30,000 for the first time.

Takagi lodged at Kreuzbergweg 15. The house is a rather small, two-storied structure. It is about seven minutes' walk from the univercity. Yoshiye later told his family: "In those days of Göttingen, the students strove to improve their fencing skills as preparation for duels. Those, who had scars on their faces, behaved themselves very haughtily." Ôgai Mori, a famous Japanese writer, also described, in his *Diary in Germany*, the duels which he observed with great interest. The prevalence of duels among German students seems to have made very queer impressions on Japanese youths who were studying in Germany.

In September of 1900, the Second International Congress of Mathematicians was held in Paris. It is well known that Hilbert presented 23 unsolved problems in this congress. Fujisawa, after attending it as the delegate of Japan, visited Germany; and in Göttingen, Takagi and Yoshiye welcomed him. Fujisawa asked them to lead the way to the grave of Gauss. However, they did not know where it was. The opinion of Takagi about it was as follows: 'It is of no use to visit the grave of Gauss. If there should be such time to spare, it would be better to study his writings.' But it was not very polite to say so to his master. With Yoshiye he searched out the grave, and led Fujisawa to it. This little episode is very interestig to the author: young Takagi was a realistic, modern youth!

Lectures by Klein

The professors in the department of mathematics were F. Klein and D. Hilbert. A. Schoenflies, F. Fischer, and F. Schilling were assistant professors, while E. Zermelo and M. Abraham were private docents.

Takagi wrote in his *Recollections and Views of the Future* as follows:

'I was much astonished by the striking contrast in the atmospheres of the mathematics departments of Göttingen and of Berlin. In the former, once a week a meeting was held, and in attendance was a group of brilliant youths from all over the world, as if here were the

center of the mathematical world. I was already at the age of twenty-five, and felt painfully that my knowledge was fifty years behind the then stage of mathematics. To recover that fifty years' lag seemed to be very difficult. But, within three semesters in the atmosphere of Göttingen, I began to feel I had almost recovered the lag. Thus I was made to realize how important atmosphere is in pursuing scientific truth.···

Klein's lectures were then very popular. By the regulations of German universities, one could attend any lecture simply if he paid the fee of the lecture he attended. In order to decide whether or not he would attend it regularly, he could attend it for about six weeks, free of fee. Now, I was very much fascinated by Klein's lectures. However, to tell the truth, I never paid the fees of his lectures; I attended them for about six weeks, and then stopped. That was quite enough. Such attendance for six weeks helped me so much to recover the fifty years' lag. I guess that attending them to the end would have hardly been effective for such a purpose. At any rate, the general surveys, stated at the beginning of the lectures, were quite interesting. Klein often used the expression "Three Large A's"; they were *Arithmetic, Algebra,* and *Analysis.* I interpreted his implicit intention as follows: "Ordinarily, one studies mathmatics, dividing it into several fields, while I unify them by my geometry." I attended those six weeks' lectures with much interest, since they were based on such a unifying spitit.'

In the culture of today, there is a very remarkable tendency to grasp the common, essential object in several, apparently quite different things. Perhaps this can be called *high essentialism.* We can find it in Klein, and more intensely in Hilbert.

Kronecker's Dream of His Youth

In 1897, Hilbert published his *Theory of Algebraic Number Fields.* Takagi seems to have read this book in Berlin. Hilbert described in it, concisely and clearly, the main (then known) results of algebraic number theory, including his own methods and results, in a certain unifying system, and suggested that the theory of relatively Abelian fields might be the most fascinating object of study in this area, containing hidden, beautiful general laws. This book played the role of the Bible in Takagi's scientific life.

In *Recollections and Views of the Future,* Takagi stated:

'When I went to consult Hilbert for study, it seems that I was looked upon by him with the eyes of suspicion. He asked me: "You say you intend to study algebraic number theory. Is it really true?" In the world of that time, that theory was being studied almost ex-

clusively in Göttingen. Thus it is no wonder that he never expected an Oriental to study it. I answered. "I intend to do so." "Then, by what object is an algebraic function determined?", he asked immediately. I could not answer at once. "It is determined by its Riemann surface," the professor himself gave the answer. Since it was true, indeed, I replied, "Oh, yes, that's right!" Perhaps he must have considered my mathematical ability to be not very reliable. Then he ordered me to accompany him, since he was on the point of going home. I told him that I intended to study *Kronecker's Dream of His Youth* in the case where the ground field was the Gaussian number field. "That's fine," said Hilbert.···'

Kronecker's Dream of His Youth was a (then unsolved) conjecture by Kronecker; any relatively Abelian field over an imaginary quadratic field K is to be obtained from transformation equations of such elliptic functions that have complex multiplications by elements of K. In his Göttingen days, Takagi could solve it affirmatively in the above-mentioned special case. After returning to Japan, he obtained a doctor of science degree with that paper. We might add that he could solve this conjecture completely after about twenty years, when he succeeded in establishing his class field theory, and applied it to the conjecture.

His stay in Göttingen has clearer significance in his scientific life than that in Berlin has. He began to do real research in this period. What gave him stimulation toward it was the *active atmosphere of creative researches*, engendered by brilliant young mathematicians from all over the world. Takagi was touched off by this!

However, Hilbert was then already distant from algebraic number theory, and thus Takagi could not get from him particular guidance in study. But Hilbert had very rare ability of grasping sharply the most essential secret, which was hidden in some complicated objects, but which dominated them really. Such high essentialism is to be seen also in Takagi's class field theory. It seems that Hilbert strongly exerted upon him the imfluence of such an attitude of mind.

IV. The peaceful days

The tall pine trees by Okua Temple

At the end of September in 1901, Takagi left Göttingen for his native country.

One of his post-cards to Yoshiye (still in Göttingen), written during this travel, is as follows;

'I send you best wishes for a Happy New Year from Hongkong on November 19. Since I will be returning home in about ten days, I am now in high spirits.···'

Thus finishing his study abroad, which had taken three years, he arrived in Kôbe on December 4. He was at that time twenty-six years old.

In the Village of Kazuya, his home, there was a custom among the villagers since olden days to see off and welcome home travellers under the five tall pine trees by Okua Temple on the outskirts of the village. Takagi came home on a jinrikisha (Japanese car drawn by a man), and, at this very place, he was welcomed with much enthusiasm by his relatives and his neighbours, and also by about three hundred pupils of Isshiki Primary School, led by the headmaster. They brought banners, on which was written in large letters 'Welcome Home, Mr. Teiji Takagi'. In those days of Japan, to study abroad was a great event!

Kansuke Takagi, however, had died before his foster son's return. On March 18 of that same year, he accidentally fell into the river from a ferryboat. Though he was rescued at once, he caught cold, which developed into pneumonia. Thus, on March 31, he passed away at the age of sixty-four.

That dandy gentleman in the house over there

Takagi began to give lectures in the Imperial University of Tôkyô as an assistant professor. (In 1897, 'of Tôkyô' was added to the name, since another Imperial University had been founded in Kyôto in that year. Those were then the only two national universities of Japan.) Takagi rented a house at Akebono-chô 13, Hongô, and was helped in domestic chores by Tsune, his mother, who had come from Kazuya.

As appropriate for a man who had just returned from abroad, Takagi used to dress in stylish clothes of excellent quality, wearing nose-glasses, and cultivating a moustache like that of Kaiser Wilkelm II of Germany. The neighbours secretly called him 'that dandy gentleman in the house over there'.

The owner of his house was named Tetsuya Mizushima, a professor of Tôkyô High School of Commerce. His wife, Mitsu, was the very woman who was later to mediate in the marriage of Takagi; for she considered him to be very desirable as her younger sister Toshi's future husband. Kuninosuke Tani, the father of Mitsu and Toshi, was a samurai (Japanese warrior) before the Meiji Restoration, but he was then the head of a county in Miye Prefecture.

In January of 1902, at the Mizushima's, Takagi formally met Toshi with a view to arranging their engagement. (Such a meeting is called *miai* in Japanese. The meaning of the word is 'looking at each other'. In those days of Japan, most marriages were arranged by miai.) At the meeting, Takagi spoke little, though he did give a minimum answer

to any question. Toshi felt him rather difficult to get to know; she was somewhat awed by him—in fact, a little frightened of him. Once she was on the point of sneezing; however, in front of this awesome gentleman, she suppressed the fit of sneezing. Though being a trifling episode, it is representative of the feelings of a young Japanese girl of a good family in the Meiji Era.

They held the wedding on April 6 of the same year. For about three months, Tsune, Takagi's mother, lived with them, and then returned to Kazuya, satisfied with this very good daughter-in-law.

The peaceful thirteen years

After his return from Germany in 1901, Takagi published five short papers in the first two years. They were not of deep originality, compared with his later papers. In 1903, he published the paper on Kronecker's Dream of His Youth, which he actually wrote in Göttingen, as we stated before. He obtained by it the degree of doctor of science on December 26 of the same year. He was promoted to full professor in the next year, at the age of twenty-nine. However, after publishing the above paper, he did not write any paper until 1914, and continued to lead a life of apparent scientific calmness.

Now, Toshi had an elder sister, Hana, who was younger than Mitsu, Mizushima's wife. Hana lived very long, indeed until only several years ago, and thus, fortunately, the author could interview her. The following description of Takagi's private life is based mainly on conversations with her.

As we mentioned before, Kuninosuke Tani, Toshi's father, was a samurai before the Meiji Restoration. He was so austere that, whenever one of his daughters was sitting (in Japanese style) even a little at ease, then he used to hit her with his long Japanese pipe, without saying a word. Even he, however, seems to have been embarrassed by the unsociability of Takagi. He once grumbled to his family: "When I am talking with Teiji, I feel quite stiff!"

Toshi was a cheerful, sociable woman. She talked with visitors very loudly. In such cases, Takagi would be sitting beside her practically silent. However, whenever she was incorrect about dates in her talks, then he used to scold her instantly.

When a person visited him for the first time, Takagi immediately grasped his disposition and habits—especially, rather his faults. He was very keen in human sensibility.

As was the natural thing for Japanese men in the Meiji Era, he was the boss in his house. Nevertheless, he often bought presents (mufflers, for instance) for his wife in Ginza (a main street of Tôkyô). His choice in such shopping was of good taste, as could be expected

of 'that dandy gentleman in the house over there'.

This couple was blessed with many children—three sons and five daughters. One of their sons, Sachio, later became a physicist, and is now a professor of Tôkyô University.

V. Class field theory

The outbreak of the First World War

In 1914, the year when the First World War broke out, Teiji Takagi began to create his class field theory; and in 1932, he held a dinner party at a hotel in Zurich, inviting about ten mathematicians (Emmy Noether, Helmut Hasse, Claude Chevalley, and others), most of whom had some connection with his theory. These eighteen years from 1914 till 1932 can be considered to be the unique gay period in the long quiet life of Takagi.

First let us quote again from his *Recollections and Views of the Future*, from which we have quoted so often:

'Generlly speaking, I am a man who cannot do anything without stimulation. Different from the circumstances of today, there were then almost no persons in Japan who were studying the same speciality as mine. Therefore, I could not get stimulation from others. It was an era when one might spend his days vacantly.

Now in 1914, the World War broke out. That was a good stimulation for me—a negative stimulation; European books stopped coming to Japan. A certain scholar was reported with ridicule in the newspapers to have grumbled that, since German books were not to be obtained, science could not be studied in Japan. In my own case, since European books did not reach me, I had to study by myself, if ever I wished to study. If the World War had not broken out, I might have achieved nothing.'

Creation of class field theory

In 1915, the second year of the War, Takagi published two short papers in German on relatively Abelian fields. He stated in them an outline of the proof of the following Existence Theorem: for any given ideal class group A/H (using congruence classes in the sense of Weber) in an algebraic number field k, there exists a relatively Abelian field K over k such that K is a class field over A/H and that both Decomposition Theorem and Isomorphism Theorem hold for K.

However, the central result in Takagi's class field theory is the next one, which he later called *Fundamental Theorem*: any relatively Abelian field K oven an algebraic number field k is a class field over some ideal class group $A_{\mathfrak{m}}/H_{\mathfrak{m}}$ in k, if one takes the modulus $\mathfrak{m}$ appropri-

ately. At first, he succeeded in proving it in the case of cyclic extension of prime degree, and then of prime-power degree. Finally, he generalized it to arbitrary Abelian extensions.

This was a great and unexpected result. Since the scientific interchange with German mathematicians had been interrupted, Takagi could not easily gain confidence about its correctness. The proof was vely long and difficult. In those days of Japan, no one other than he could check the proof. In *Recollections and Views of the Future*, he wrote:

'Since it was then quite an unexpected result, I thought it was surely incorrect. So I looked for where it was wrong. Perhaps, I was then a little neurotic. I often had dreams concerning the study. In a dream, I would believe that I had solved a problem, and then, waking up, I would actually work it out at the desk, finding alas it was quite mistaken. I looked for counter-examples in vain. I had been searching for mistakes for a long while. I had hardly any confidence in my result even after I constructed the theory. If there is a hole made by ants in some place, the whole structure collapses due to that small hole. At least in mathematics, we cannot accept what is only roughly correct.'

Isao Takagi, Teiji's eldest son, related to the author the following recollection: in 1920, when he Isao was a fifth-year pupil in primary school, he was once playing in his house, not so noisily. Suddenly, his father rushed out from his study, and hit him on the head with a magazine, shouting, "Don't be noisy!" It was not painful, but it astonished the boy considerably, since it was so sudden, and moreover, he was not making much noise. This episode shows the great strain of Takagi, who was then on the point of completing a magnificient theory in utter scientific solitude.

In this theory, Takagi grasped excellently, by analysis of high level, the common essence which lies hidden in apparently quite different objects—theory of quadratic forms by Gauss, Dirichlet's Theorem on primes in arithmetical progressions, Weber's Theorem (any Abelian field over the field of rational numbers is contained in a cyclotomic field), Furtwängler's studies on absolute class fields, and so on. And he could even solve affirmatively that Kronecker's Dream of His Youth, which, in the general case, had been pending since his days in Göttingen, applying his class field theory. Thus through his very utmost effort, he completed his highest achievement.

Strasbourg (1920)

The First World War came to an end in 1918. In 1920, Takagi got a chance for his new theory to be checked by foreign mathemati-

cians: he was ordered by the Ministry of Education to observe foreign universities after the War. Since the International Congress of Mathematicians was to be held in Strasbourg, he intended to publish his theory there. He wrote down all of his results in a very long paper (133 pages) in German entitled 'On a theory of relatively Abelian number fields'.

He started from Kôbe on July 8 of the year, and reached Marseilles about August 20. He spent about a month in Paris, and then went to Strasbourg.

The congress was begun on September 22. At the reception in the evening of that day, a European mathematician whispered to others: "I hear that Japanese fellow will talk on number theory. Perhaps, it must be Fermat's Problem. We shall surely be amused!" The whisper reached Takagi's ears, and he smiled a bitter smile.

On the fourth day of the congress, Takagi delivered his address in French, entitled 'On several general theorems in the theory of algebraic number fields'. The chairman was L. E. Dickson. The room was a rather small one, and the audience comprised about fifty persons, including Fueter, Hadamard, Châtelet, Eisenhart, and D. E. Smith. Takagi did not use the blackboard. After the address, no one asked him any questions. Only a very few of those present (perhaps, Fueter and Hadamard) seemed to have understood a little of what he had said. The German mathematicians, some of whom might have understood better, had all been shut out from this congress, due to the strong hatred for the Germans. Thus, as a whole, Takagi could not get any distinct reaction from this address.

After the congress, he returned to Paris, and then visited some European countries. Since he was very found of *Travel in Italy* by Goethe, he much enjoyed his own travelling in that country, especially, a stay in Florence. In Göttingen, he visited his old master Hilbert. He heard in this city that, among the young mathematicians there, there was a brilliant person, named Siegel, who was studying number theory. After visiting the United States, he returned to Japan on May 13 of 1921, perhaps in disappointment.

The appearance of Emil Artin

Takagi's life returned again to the former peaceful pattern. Until 1925, when Helmut Hasse in Germany delivered an address on class field theory, almost none of the mathematicians of the world knew the name of Takagi, nor had any concern about him.

In 1922, Takagi published a paper entitled 'On the reciprocity law in arbitrary algebraic number fields'. In it, using his class field theory, he proved in a much simpler and prettier way Furtängler's reciprocity

law on residues modulo powers of l (an odd prime), and formulated the essential part of this theorem in such a form as might suggest Artin's reciprocity law. This paper is now called the Second Paper, his paper in 1920 the First Paper.

Emil Artin was born in Vienna in 1898, the son of an Armenian art-dealer and an operetta singer-actress. Immediately after he began to study mathematics in Vienna University, he was drafted into the army, owing to the War. But he did not serve in any actual battle, instead working as an interpreter of French. After the War, he studied in Leipzig University, and obtained the degree of doctor in 1921. For a year after that, he was studying in Göttingen.

Now we mentioned before that Takagi heard in Göttingen about the brilliancy of Siegel. After returning home, he sent a reprint of his First Paper to Siegel.

One day, when Siegel was talking with Artin about algebraic number fields, he took out the reprint which Takagi had sent to him, and persuaded Artin to read it. This was at the beginning of 1922.

Artin borrowed the reprint from Siegel. He spent three weeks in reading it through. Later, in 1962, he told the present author: "I felt strong admiration for it. It was not difficult to understand, since it was written very clearly."

In 1923, the idea occurred to Artin to formulate the most general type of reciprocity law, using Frobenius substitutions. It was suggested to him by Takagi's Second Paper. He called it the *General Reciprocity Law*, and he could prove it in several special cases. But he was quite in the dark when it came to proving it for the general case.

Here we should like to add a word about Ernst Hecke. He also acknowledged the high value of Takagi's two papers. In his book *Lectures on the Theory of Algebraic Numbers* (1923), he wrote as follows: 'Recently, on a line near that of Hilbert, Takagi has made some very fruitful contributions, by which we are able to have a complete survey on all relatively Abelian fields, having similar relations as cyclotomic fields over the field of rational numbers.' He also reviewed the two papers in detail in *Fortschritte der Mathematik*.

Young Helmut Hasse

On September 15 in 1925, five years after the Congress in Strasbourg, Helmut Hasse, Professor of Halle University, of age twenty-seven, presented an outline of Takagi's theory at the annual meeting of the German Mathematicians' Union, held in Danzig. His address was entitled 'Recent progress in class field theory', and about one hundred persons attended the lecture. The following year, he began to publish a series of detailed reports, entitled 'Report on recent re-

searches and problems from the theory of algebraic number fields', in three separate parts (in 1926, 1927, and 1930), in the Annual Report of that Union, describing Takagi's theory and Artin's Reciprocity Law with complete proof, arranged in his own way. The address and report by Hasse played a vital role in presenting Takagi's class field theory to the mathematicians of the world.

Hasse was born in 1898, as a son of a judge in the City of Kassel. He began to acquire a liking for mathematics somewhere between the age of fifteen to seventeen, when he thought that it was a pure science, and moreover, he felt in it a thrill in researching the unknowns as in detective stories.

By the time Hasse graduated from gymnasium, Germany had already been involved in the World War. He entered the Navy, and engaged in several battles, for instance, a sea battle with Russia on the sea between Sweden and Finland. The warships on which he served changed several times, but he was aboard the Strasbourg and the Augusburg, both cruisers, for a rather long time.

In the course of time, he was transferred from warships, and served in Kiel, a naval port, as a member of the staff in the Center of Naval Operations. During this period, though the War was still going on, he began to study mathematics by himself. After the War, he entered Göttingen University, and attended the lectures by Landau and Hecke. In 1920, he moved to Marburg, where he studied under Hensel, and was awarded the degree of doctor in 1921.

In 1923, Artin urged him to read the two papers of Takagi. Reading the First Paper, Hasse was deeply fascinated by its generality, its clearness, its effective methods, and its wonderful results. He was given an even stronger inspiration by the Second Paper.

He completely analyzed the contents of the papers, and endeavored to put in order and make clear, in his own way, the long and difficult proofs of Takagi's theorems. It was when he completed this work that he began to present the theory to the mathematical world, as we mentioned before.

In 1962, the author had an interview with Hasse in Hamburg. His eyes seemed just to fit the adjective *ernst* in German. On a wall of his study, there were hung about ten photographs of those mathematicians he respected—Hensel, Hilbert, and others. And as many as two photographs of Takagi were there.

VI. The dinner party in Zurich

Tschebotareff, Herbrand and Chevelley

In 1926, Tschebotareff of Kazan University proved the *Density*

Theorem concerning normal extensions of algebraic number fields. In its proof, he employed an original method—*crossing by cyclotomic fields.* Artin was much interested in this new method; and the following year, in 1927, he finally succeeded in proving his General Reciprocity Law, using this method. The theorem contained many known reciprocity laws—those by Gauss, Eisenstein, Kummer, Hilbert, Furtwängler, and others. Thus it was to be considered a decisive form of reciprocity laws, and one could derive from it, easily and concretely, the Isomorphism Theorem and Decomposition Theorem in Takagi's class field theory.

On February 26th in 1929, Professor A. Guldberg of Oslo University wrote a letter to Takagi: 'On April 6th, the University will hold a festival in celebration of the Centenary of the death of Niels Henrik Abel, which falls on that date. On the occasion you will be created a doctor honoris causa of the University. The Rector of the University desires me to enquire whether you are able to be present at this commemorative festival.' However, since it then took many days to travel to Europe from Japan, Takagi could not attend the festival. The other mathematicians, who were also awarded the same degree on this occasion, were E. J. Brouwer, F. Engel, R. Fueter, J. Hadamard, G. H. Hardy, K. Hensel, C. Juel, E. Landau, E. L. Lindelöf, P. Painlevé, L. E. Phragmén, S. Pincherle, de la Vallée Poussin, O. Veblen, and H. Weyl.

In 1930, applying his General Reciprocity Law, Artin reduced Hilbert's *Principal Ideal Theorem* (then yet conjecture) to a purely group-theoretic assertion. Furtwängler, who had been a faithful disciple of Hilbert in algebraic number theory, and who had proved Hilbert's conjectures on absolute class fields, succeeded, in the same year, in proving Principal Ideal Theorem, based on Artin's reduction to group theory. It created a sensation at the time in the mathematical world. Shôkichi Iyanaga, a young disciple of Takagi, generalized this result to class fields of a certain type also in 1930. (Later in 1934, Iyanaga gave a simple proof of Principal Ideal Theorem.)

On July 27 in 1931, Jacques Herbrand died suddenly from an accident in mountain-climbing, at the age of twenty-three. When Takagi proved the Fundamental Theorem of his theory, he first proved it for the case of extensions of prime degree, and then shifted to the case of prime-power degree. The shift was done complicatedly, using the theory of genera. Herbrand had simplified it remarkably by a lemma, which was group-theoretical, and which is now called *Herbrand's Lemma.*

Iyanaga went to Europe in 1931, and studied under Artin in Hamburg, where he became intimate with Claude Chevelley from

France. One day, Chevalley astonished Iyanaga by saying, "I have succeeded in proving class field theory arithmetically." In Takagi's theory, a certain important inequality was proved by an analytical method, using L-functions. The above adverb 'arithmetically' means without analysis. Actually, Chevalley's proof then was not completely arithmetical, but it was quite ingenious, using the crossing method. And later he wrote it in the form of a thesis, and published it in 1933 in the *Journal of the College of Science, Imperial University of Tôkyô*, in which Takagi's two main papers had also been published. He succeeded in complete arithmetification in 1940.

Olga Taussky studied in Vienna University under Furtwängler. She felt deep beauty in Takagi's First Paper, and came to respect him earnestly. She wished to talk with him in Japanese, if she might meet him. Thus she even began to learn Japanese! She investigated Principal Ideal Theorem in its capitulation process, and published a paper about it in 1932.

Emmy Noether's direct connection with class field theory was not very great. As is well known, she was one of the main leaders of abstract algebra. In 1933, she published a paper, generalizing *Principal Genus Theorem* in class field theory. However, a more important relation between her and Takagi's theory is the following: in her attitude of mind in study, there was a kind of grasping of the essence of objects in the most transparent form, not by calculations, but by the concepts of abstract algebra. This attitude greatly influenced young mathematicians, and thus contributed, even after her death, to the process of changing the classical Takagi-Artin's class field theory into the modern one, using homological methods.

Her young friend, van der Waerden, had no papers in class field theory, but he published his famous books *Modern Algebra* I and II in 1930 and 1931, based on the lectures by Artin and Noether. Here we have written about him rather because he was also one of the guests at Takagi's dinner party in Zurich.

The third trip abroad (1932)

In 1932, Takagi was ordered to visit European countries as a delegate of the Science Council of Japan. He started on July 12, and returned on December 3. It was his third and last trip abroad.

The main object of this trip was to attend the International Congress of Mathemacicians, held in Zurich, from September 4 to 12. We shall first describe his travel after the Congress.

In Vienna, Olga Taussky welcomed him, and invited him several times to dinner in her apartment, according him a warm reception with her mother and sister. She also introduced him to Furtwängler,

her master.

In Hamburg, Iyanaga introduced him to Artin, and Takagi impressed Artin as being a modest, but great scholar. Natascha, Artin's wife, wrote later in a letter to the author: 'I liked Takagi very much.' Though he was not very sociable, he seems to have been received favorably by ladies.

One day, along the road around Lake Alster in Hamburg, he strolled together with Iyanaga, tasting the atmosphere of autumn. He was then of age fifty-seven, while Iyanaga was twenty-six.

In Göttingen, Takagi, accompanied by Emmy Noether, visited his master Hilbert, who was struggling with a disease of the liver. Takagi wrote of this visit in a very interesting essay, remarking: 'Observing my old master grumbling as if speaking to himself, I wept in my heart.'

Now we shall present an account of the Congress.

As we mentioned before, R. Fueter had attend the address by Takagi in Strasbourg. He had also contributed to the theory of complex multiplications. At the Congress of 1932, he was the president, and Takagi was one of the vicepresidents. He praised Takagi in his general address 'Ideal Theory and Function Theory'.

At this Congress, an award called the Fields Medal was decided to be established, with five judges including Takagi. The president of the judging committee was F. Severi.

The dinner party

During the period of the Congress, Takagi was staying in the Hotel Eden by Lake Zurich.

Between the street for motor-cars in front of the hotel and the shore of the lake, there is a broad street for strolling, with three lines of chestnut trees. On the first floor of the hotel, there is a room for small banquets. This is the very room where Takagi held a dinner party, inviting a number of mathematicians, most of whom had some connection with class field theory.

The party was held perhaps on September 11 or 12. The guests were as follows: C. Chevalley, H. Hasse, S. Iyanaga, Y. Mimura and his wife Takako, M. Moriya, M. Nagumo, E. Noether, O. Taussky, N. G. Tschebotareff, and B. L. van der Waerden.

According to Taussky, there was a remarkable contrast between the Europeans and the Japanese; the former talked much and the latter little. She guessed that there might be a custom in Japan to be silent at table.

The person who talked most in this dinner was Emmy Noether. She was fat, indifferent to clothes, and talked very loudly. When

Tschebotareff was speaking leisurely, like a Russian, she interrupted him, throwing her hands straight forward in a gesture meaning 'stop' and then she spoke herself.

Hasse talked about his engagement in battles in the (First) World War. He said to Tschebotareff jokingly: "If the War had been going on further, I would have marched upon your city."

Throughout the evening, Takagi did not speak much. But he seemed to be respected deeply by all.

When the party came to an end, Noether, who did most of the chattering during the evening, requested the Japanese to teach her the way of Japanese bowing. She asked, bending forward her fat, upper body: "Like this, or more?" Her funny look remained long in the memories of the young Japanese mathematicians.

The dinner party seems to have been the best time in Takagi's long life. It is said that he wrote about this party in a letter to his wife in Japan, stating that he had specially selected the wines to be served.

Only two months after that evening, the Nazis became the No. 1 party in Germany. In January of the next year, 1933, the cabinet of Adolf Hitler was formed, and then the persecution of the Jews was begun. Since Noether was also a Jew, she went to the United States. In April of 1935, she was operated on for a tumor. The operation seemed to have been successful; but soon thereafter came a sudden turn, ending in her tragic death. In 1947, Tschebotareff, too, passed away. However, the other guests at that party are all well today. Especially, Hasse, aged seventy-seven, has just been accorded the joyful tribute of the publication of his entire collection of mathematical papers in three volumes.

Taussky, now named Olga Taussky-Todd, went to England in 1934, and then to the United States in 1947. After the Second World War, the first letter from abroad to Takagi was one from Taussky, inquiring after his safety.

His last years

In 1936, Takagi retired from the Imperial University of Tokyo, and, in 1940, the Order of Culture was conferred upon him by the Japanese Government. He wrote in Japanese several books of mathematics, among which, above all *Introdution to Analysis* and *Algebraic Number Theory* were very excellent ones. (The latter includes his class field theory in classical form.)

In 1951, the *Journal of the Mathematical Society of Japan* published a Commemorative Issue dedicated to Takagi, containing the papers of Artin-Tate, Brauer, Chevalley, Hasse, and Weil. However, the follow-

ing year, he suffered a personal misfortune in the loss of his wife, Toshi, who died of cancer of the lungs.

During his last several years, he enjoyed reading the Japanese classics, which he had read when he was young.

Takagi passed away peacefully on February 28 in 1960, at the age of eighty-four. His death was caused by a combination of cerebral hemorrhage and softening of the brain. There were about one thousand in attendance at his funeral. Fourteen years later, in 1974, a book *The Collected Papers of Teiji Takagi* was published.

There were no dramatic events in the long life of Takagi. However, rich in interesting connections with so many people (foremost of all, perhaps, the three generations of Kansuke), and often peculiar in cause-and-effect relationships (the leeches, the First World War, to name only two), his life entertains us fully with subtle savour.

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